The effect of a power-law mantle viscosity on trench retreat rate

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SUMMARY

The subduction of lithospheric plates is partitioned between subducting plate motion and lateral slab migration (i.e. trench retreat and advance). We use 3-D, dynamic models of subduction to address the role of a power-law mantle viscosity on subduction dynamics and, in particular, rates of trench retreat. For all numerical models tested, we find that a power-law rheology results in reduced rates of trench retreat, and elevated slab dip angles, relative to the equivalent isoviscous mantle model. We analyse the asthenospheric pressure distribution and the style of mantle flow, which exhibits only limited variability as a function of mantle rheology, in order to compute estimates of the mantle forces associated with subduction. The inclusion of a power-law rheology reduces the mantle shear force (which resists subducting plate motion) to a greater degree than it reduces the dynamic pressure gradient across the slab (which resists trench retreat). Therefore, the inclusion of a power-law mantle rheology favours a shift towards a subduction mode with a reduced trench retreat component, typically a relative reduction of order 25 percent in our 3-D models. We suggest that this mechanism may be of importance for reducing the high trench retreat rates observed in many previous models to levels more in line with the average subduction partitioning observed on Earth at present (i.e. trench velocity ≤ plate velocity), for most absolute plate motion reference frames.

Key words: Mantle processes; Subduction zone processes; Dynamics of lithosphere and mantle; Rheology: mantle.

1 INTRODUCTION

The subduction of negatively buoyant slabs constitutes the primary driving force of plate tectonics (e.g. Jacoby 1973; Forsyth & Uyeda 1975). In addition to subducting plate (SP) motion, subduction can also occur via lateral migration of the SP hinge/trench, in a deep mantle, absolute plate motion (APM) reference frame. Lateral slab migration typically occurs in the SP direction, that is, `slab rollback' or `trench retreat', but advancing trench motion towards the overriding plate (OP) also occurs at a limited number of trenches (e.g. Elsasser 1971; Carlson & Melia 1984; Jarrard 1986). While the choice of absolute reference frame complicates the deduction of trench migration rates and directions (e.g. Heuret & Lallemand 2005; Funiciello et al. 2008; Lallemand et al. 2008; Schellart et al. 2008; Becker et al. 2015), trench migration rates (V_T) are, on average, less than ~50 per cent of total convergence rates (V_C), albeit with large spread (e.g. Schellart et al. 2008; Becker & Faccenna 2009). SP motion (V_P) accommodates the remainder of the subduction rate, that is, V_C = V_P + V_T. The rates and direction of trench motions have direct consequences for the style of back arc deformation (e.g. Uyeda & Kanamori 1979; Lallemand & Heuret 2005), and understanding `typical' V_T/V_C ratios is a prerequisite for using them to constrain plate reconstructions, including their APM component (e.g. Williams et al. 2015).

Numerical and analogue subduction models, within which the trench migrates self-consistently (i.e. `dynamic models'), have been used extensively to explore the mechanical controls on SP kinematics. The total convergence velocity can, to first order, be predicted by considering the interplay of the slab pull driving force and the resisting forces associated with slab bending and mantle drag (e.g. Conrad & Hager 1999; Bellahsen et al. 2005). However, what dictates how convergence partitions between SP, V_P, and trench motion, V_T, remains difficult to isolate. Modelling studies have typically focused on isolated subducting slabs, neglecting the effect of an OP. These studies highlight, in particular, the strong control of slab strength on V_T (e.g. Enns et al. 2005; Bellahsen et al. 2005; Di Giuseppe et al. 2008; Funiciello et al. 2008; Faccenna et al. 2009). In addition to being consistent with observations relating to plate flexure (Billen & Gurnis 2005), plate-mantle viscosity contrasts of 100–1000 are generally required to produce both the variability in trench migration regimes, and average subduction partitioning (i.e. typically V_T ≤ V_P) inferred to occur on Earth (e.g. Schellart 2004; Funiciello et al. 2008; Becker & Faccenna 2009). However, even for slabs within this intermediate strength range, single slab studies often appear to favour trench motion over plate motion, to a greater degree than that observed on
Earth (e.g., Funiciello et al. 2003; Stegman et al. 2006; Schellart et al. 2007).

For a given slab strength, a significant trench retreat reducing feature of the subduction system appears to be the presence of an OP. Dynamic models that incorporate an OP are now routinely studied in 2-D (e.g., Capitanio et al. 2010; Sharples et al. 2014; Garel et al. 2014; Holt et al. 2015a; Petersen et al. 2015), and increasingly so in 3-D (e.g., Yamato et al. 2009; Duarte et al. 2013; Schellart & Moresi 2013; Jagoutz et al. 2015). The presence of an OP modifies the force balance acting on the SP, which shifts subduction partitioning toward SP motion, that is, reducing $V_T / V_p$ (Yamato et al. 2009; Butterworth et al. 2012; Holt et al. 2015a). Additionally, in 3-D, the width of the trench has been shown to exert a significant control on rates of trench retreat, with wide trenches typically exhibiting reduced rates in numerical models, and on Earth (e.g. Dvorkin et al. 1993; Stegman et al. 2006; Schellart et al. 2007; Stegman et al. 2010). Here, we focus on the role that a stress-dependent mantle rheology, a feature typically neglected in dynamic subduction models, has on the partitioning of subduction between $V_T$ and $V_p$.

The 3-D modelling studies discussed above typically consider a uniform viscosity mantle, and so neglect the role of lateral viscosity variations in the sublithospheric mantle. Laboratory experiments, however, predict that deformation in large regions of the upper mantle is dominated by dislocation creep, in which the effective viscosity has a power-law dependence on stress (e.g., Karato & Wu 1993; Hirth & Kohlstedt 2003). Furthermore, seismic anisotropy observed in the upper mantle can likely be attributed to dislocation creep deformation, which generates lattice preferred orientations in olivine (e.g. Nicolas & Christensen 1987). Analyses which combine observations of seismic anisotropy with global, or regional, flow computations suggest that flow in the oceanic mantle is dominated by dislocation creep between depths of ~100 and 400 km (e.g. Podolefsky et al. 2004; Becker 2006; Behn et al. 2009). 2-D numerical models show that the inclusion of a stress-dependent rheology decreases the mantle wedge viscosity, which reduces the hydrodynamic stresses on the upper surface of the slab, thereby resulting in an increased slab dip angle (Tovish et al. 1978; Billen & Hirth 2005, 2007). However, these models fix the trench location and so uncertainty remains regarding whether variations in trench rollback rate, which exert a first order control on slab dip (e.g. Christensen 1996), would counteract this dip-increasing effect. A number of 2-D, numerical models that do allow the trench to move freely have been used investigate how the slab rheology influences slab kinematics and slab-interaction with a more viscous lower mantle (e.g., Androvićová et al. 2013; Čížková & Bina 2013; Garel et al. 2014; Holt et al. 2015a), but the role of a stress-dependent mantle viscosity on trench rollback has not, to our knowledge, been explicitly considered in a dynamic framework. While relative reductions in mantle viscosity triggered by a stress-dependent viscosity intuitively result in elevated slab sinking rates, and greater subduction-induced mantle flow velocities (e.g. Jadamec & Billen 2012; Billen & Jadamec 2012), the degree to which such localized weakening shifts the convergence partitioning between $V_T$ and $V_p$ is not clear.

Coupled to the subduction kinematics is the style of subduction-induced mantle flow. Subduction excites poloidal flow in the vertical plane and, in 3-D, toroidal flow. Return flow associated with a near-stationary trench is dominantly poloidal, whereas horizontal motion of a subducting slab, in the form of trench retreat or advance, promotes toroidal flow around the slab edges (e.g., Dvorkin et al. 1993; Butlles & Olson 1998; Kincaid & Griffiths 2003; Funiciello et al. 2003, 2004; Schellart 2004; Stegman et al. 2006; Piromallo et al. 2006). Understanding the dynamic controls on the style of subduction-induced return flow is important both in aiding the interpretation of shear-wave splitting measurements which can probe horizontal/subhorizontal mantle flow (e.g. Long & Becker 2010), constraining the mantle pressure field and the intertwined slab kinematics (e.g. Royden & Hussen 2006), and understanding the locations of off-axis volcanoes associated with subduction-induced upwelling flow (e.g. Facenna et al. 2010). Therefore, we also analyse the mantle flow regime in our numerical models. After characterizing how the presence of an OP affects mantle flow in a dynamically evolving subduction setting, as previously considered for 3-D numerical models which prescribe slab kinematics (Honda 2009), we assess whether the inclusion of a power-law mantle rheology modifies the style of subduction-induced mantle return flow.

2 METHODOLOGY

We use the finite-element code CitcomCU (Moresi & Gurnis 1996; Zhong 2006) to construct numerical, time-evolving subduction models in both (pseudo) 2-D and 3-D domains. CitcomCU solves the conservation equations which govern convection in an incompressible viscous fluid (Boussinesq approximation) with negligible inertia. Our models solve for the conservation of energy, mass (continuity eq.), and momentum (Stokes eq.), but we neglect thermal diffusion (see Section 2.1). 2-D models are less computationally expensive and so allow us to initially probe the parameter space more extensively in order to inform our 3-D model parameters, as well as examine the role of lithospheric viscoplasticity. 3-D models are then used to verify the applicability of the 2-D model systematics to subduction systems with finite width trenches. In addition to plate kinematics, the 3-D models are used to investigate the dynamic evolution of both the mantle flow and viscosity fields. The mechanical and rheological parameters are equivalent for the two suites of models which initiate with a deep initial slab (see Fig. 1 and Table 1). Numerically, the only differences are that the 2-D models contain only one finite element in the third, y-dimension (e.g. Holt et al. 2015b), while 3-D models contain many elements (256) in order to resolve a finite width trench and a passive side plate. That aside, the two models contain the same sized and number of finite elements in the vertical plane. Our explanation of the 3-D model setup in the next section is therefore also applicable to the 2-D models presented in section 3.1.

2.1 Numerical model setup

In order to target the first order effects of a power-law rheology on slab dynamics, we use a simplified geometrical, mechanical, and rheological setup. We consider two active plates, an SP and an OP, and a passive side plate which is separated from the subduction system plates by 25 km of mantle material. The side plate, while not participating directly in subduction, prevents rapid plate shortening in the trench parallel direction, and spontaneous subduction along the edge of the SP. Without the side plate, such unphysical effects occur due to the lateral juxtaposition of dense lithosphere with light mantle, a configuration not expected to occur on Earth (e.g. Yamato et al. 2009). The side plate also provides a convenient reference frame for velocities measured in the subduction systems (e.g. trench retreat rates). The trench-perpendicular lengths of the SP and OP exert a strong control on rollback rates, and so we examine variable plate lengths, relative to the vertical (upper mantle) dimension of 660 km, to assess the effect of a power-law viscosity on models with a wide range of slab kinematics.
We simplify our model setup by neglecting thermal diffusion (e.g. Enns et al. 2005), thereby considering a density and viscosity field that is dependent purely on composition, which is advected using tracer particles (McNamara & Zhong 2004). The effect of thermal diffusion to disperse slabs is dependent on the local Peclet number, which is typically large. Thermal slabs are therefore expected to be fairly coherent in the lower mantle (e.g. Tan et al. 2002), and so we consider this a reasonable simplification to enable extraction of the core physical processes. The domain size is 7920 km (length, $x$) $\times$ 660 km (height, $z$) $\times$ 5280 km (width, $y$), and all boundaries are mechanically free slip. We initially place 40 tracers within each of the elements, which are used to define the lithospheric plates, and the weak ‘crustal’ layer that decouples the lithospheric plates. The finite element dimensions are $\approx 14$ km. A free-slip boundary condition may render the slab tip overly mobile atop the upper-lower mantle boundary, and so we additionally test the effect of a no-slip lower boundary which approximates a high viscosity lower mantle.

We consider plates with widths of 2000 km. For each of the 3-D models, we take advantage of the plate symmetry about $y = 0$ (i.e. trench-perpendicular horizontal axis), which enables us to half the computational domain (i.e. $y = 2640$ km). Lithospheric plates have a constant density contrast, relative to the sublithospheric mantle, of 85 kg m$^{-3}$ and a constant thickness of 80 km. The sublithospheric mantle has a reference density of 3300 kg m$^{-3}$. Subduction is initiated by allowing the SP to extend below the OP with a centre-line radius of curvature ($R_{\text{init}}$) of 150 km. The centre-line is the line which is equidistant from the two surfaces of the SP (Fig. 1). Below the curved portion of the initial lithospheric geometry, we allow the plate to extend to a depth of 500 km with an initial dip angle of $70^\circ$. The initial, deep placement of the slab-tip near the model base results in a suite of models which have a consistent style of evolution, with all slabs bending backward on top of the impermeable base of the domain, as opposed to slabs which bend forward and roll over the lower boundary. While this initial condition reduces the observed variability in slab kinematics, it allows us to focus on the subduction mode which appears most akin to subduction on Earth. Otherwise, much of the kinematic variability after slab interaction with the model base is a direct result of the different styles of slab bending. However, for the 2-D models, we do examine a suite of models in which the initial slab is restricted to shallow depths of <150 km.

### 2.1.1 Rheology

We use a composite Newtonian (diffusion creep) and power-law (dislocation creep) rheology (e.g. Zhong et al. 1998). Due to the compositional nature of our models, we neglect the temperature,

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**Figure 1.** Schematic illustration of the initial conditions for the 3-D numerical models. The 2-D numerical models have an equivalent setup but neglect the $y$-dimension (i.e. dimension with length = 2640 km).

**Table 1.** Numerical model parameters.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>Domain height ($z$)</td>
<td>660</td>
<td>km</td>
</tr>
<tr>
<td>$l$</td>
<td>Domain length ($x$)</td>
<td>7920</td>
<td>km</td>
</tr>
<tr>
<td>$w$</td>
<td>Domain width ($y$)</td>
<td>5280</td>
<td>km</td>
</tr>
<tr>
<td>$w_T$</td>
<td>Trench width</td>
<td>2000</td>
<td>km</td>
</tr>
<tr>
<td>$h_{\text{lith}}$</td>
<td>Lithospheric thickness</td>
<td>80</td>
<td>km</td>
</tr>
<tr>
<td>$h_{\text{crust}}$</td>
<td>Crustal thickness</td>
<td>15</td>
<td>km</td>
</tr>
<tr>
<td>$L_{\text{SP}}$</td>
<td>Subducting plate length</td>
<td>2000, 3000, 4000</td>
<td>km</td>
</tr>
<tr>
<td>$L_{\text{OP}}$</td>
<td>Overriding plate length</td>
<td>2000, 3000, 4000</td>
<td>km</td>
</tr>
<tr>
<td>$R_{\text{init}}$</td>
<td>Initial radius of curvature</td>
<td>150</td>
<td>km</td>
</tr>
<tr>
<td>$z_{\text{notch}}$</td>
<td>Initial slab depth</td>
<td>150, 500</td>
<td>km</td>
</tr>
<tr>
<td>$\theta_{\text{init}}$</td>
<td>Initial slab dip angle</td>
<td>70°</td>
<td></td>
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### Mechanical and rheological properties

<table>
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<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
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<tbody>
<tr>
<td>$\rho_0$</td>
<td>Mantle density</td>
<td>3300</td>
<td>kg m$^{-3}$</td>
</tr>
<tr>
<td>$\Delta \rho$</td>
<td>Plate-mantle density contrast</td>
<td>85</td>
<td>kg m$^{-3}$</td>
</tr>
<tr>
<td>$\eta_{\text{lith}}$</td>
<td>Lithosphere viscosity</td>
<td>$1.4 \times 10^{23}$</td>
<td>Pa s</td>
</tr>
<tr>
<td>$\eta_{\text{crust}}$</td>
<td>Crust viscosity</td>
<td>$2.8 \times 10^{20}$</td>
<td>Pa s</td>
</tr>
<tr>
<td>$\eta_{\text{N}}$</td>
<td>Newtonian, mantle viscosity</td>
<td>$2.8 \times 10^{20}$</td>
<td>Pa s</td>
</tr>
<tr>
<td>$n$</td>
<td>Power-law exponent</td>
<td>3.5</td>
<td>–</td>
</tr>
<tr>
<td>$\dot{\varepsilon}_T^{\text{II}}$</td>
<td>Transition strain rate</td>
<td>$10^{-15}, 5 \times 10^{-15}, 10^{-14}, 10^{-13}$</td>
<td>s$^{-1}$</td>
</tr>
</tbody>
</table>
dependence of viscosity, and additionally omit the pressure dependence. As reviewed recently by King (2016), there remains significant uncertainty regarding the appropriate creep law parameters, even for the dominant upper mantle mineral, olivine. Additionally, grain size, which exerts a strong control on the relative dominance of diffusion and dislocation creep, is likely both spatially variable and time-dependent. Given such uncertainties, we therefore chose a simplified rheological parameterization, which enables us to isolate first order dynamic effects, and ensures relatively easy reproducibility. However, the extent-depth of power-law viscosity dominance in our models is broadly consistent with the ~100–400 km inferred from studies which incorporate seismic anisotropy (e.g. Podolefsky et al. 2004; Becker 2006; Behn et al. 2009), which gives us confidence in the applicability of our rheological approach to Earth.

Our effective viscosity is composed of an isoviscous Newtonian component, \( \eta^N \), and a power-law component, \( \eta^p \), and is computed as,

\[
\eta^{\text{eff}} = \frac{\eta^N \eta^p}{\eta^N + \eta^p} = \left( \frac{1}{\eta^N} + \frac{1}{\eta^p} \right)^{-1}.
\]  

(1)

The Newtonian viscosity component in the asthenospheric mantle \( \langle \eta_{\text{mantle}}^{N} \rangle \) is \( 2.8 \times 10^{20} \) Pa s, and lithospheric plates are a factor 500 stronger than the Newtonian mantle component (i.e. \( \eta_{\text{lab}}^{N} = 1.4 \times 10^{17} \)), in line with the constraints outlined in the introduction. We impose a minimum effective viscosity cut-off of \( 2.8 \times 10^{18} \) Pa s (i.e. \( 0.01 \times 10^{-2} \gamma_{\text{mantle}}^{N} \)). We have tested a lower viscosity cut-off of \( 0.001 \times 10^{-2} \gamma_{\text{mantle}}^{N} \) for our most non-linear 2-D models, and find that the results are near identical, that is, plate velocities differ within a single per cent, and model behaviour is consistent. This viscosity formulation constitutes two drops in series, that is, \( \ddot{\varepsilon}_{\text{total}}^{N} = \dot{\varepsilon}_{\text{p}}^{N} + \ddot{\varepsilon}_{\text{p}}^{N} \), where \( \dot{\varepsilon}_{\text{p}} = \sqrt{\dot{\varepsilon}_{ij} \dot{\varepsilon}_{ij}} / 2 \) is the second invariant of the deviatoric strain rate (Einstein summation convention). The power-law viscosity component is given by

\[
\eta^p = \eta^N \left( \frac{\dot{\varepsilon}_{\text{p}}}{\dot{\varepsilon}_{\text{p}}^{N}} \right)^{1/\nu},
\]  

(2)

where \( \nu \) is the power-law exponent (= 3.5), and \( \dot{\varepsilon}_{\text{p}} \) is the transition strain rate. Due to the strain-rate dependence of viscosity, we iteratively solve Stokes equation until the global change in velocity, normalized by the absolute velocity, is below 0.01. For the 2-D case, we have verified that a stricter, power-law convergence criterion of 0.001 does not affect the model results. \( \dot{\varepsilon}_{\text{p}}^{N} \) is the strain rate at which the Newtonian and power-law components are equal (i.e. \( \dot{\varepsilon}_{\text{p}}^{N} = \dot{\varepsilon}_{\text{p}}^{N} \)). We therefore focus on the effect of varying \( \dot{\varepsilon}_{\text{p}}^{N} \) which controls the proportion of the domain within the power-law deformational regime. Because strain rates are significantly larger in the flowing mantle than in the lithospheric plates, power-law deformation is focussed into the sublithospheric mantle. Such power-law focussing is consistent with a number of modelling studies that include laboratory derived olivine creep laws (e.g. Billen & Hirth 2005, 2007; Čižková & Bina 2013).

In the 2-D suite of models, we additionally test the effect of viscoplastic yield. In this case, an additional viscosity component, \( \eta^{\text{yield}} \), is computed from a prescribed yield stress, \( \tau^{\text{yield}} \), as follows (e.g. Schott & Schmeling 1998),

\[
\eta^{\text{yield}} = \frac{\tau^{\text{yield}}}{2\sigma_f},
\]  

(3)

with a Byerlee (1968) type yield stress, \( \tau^{\text{yield}} = (a\sigma_f + b) \). \( a \) is the friction coefficient (0.6), \( b \) is the cohesion (60 MPa), \( \sigma_f \) is the lithostatic pressure, and \( \lambda \) is a constant, ‘pore pressure’ pre-factor (0.15), with a value comparable to that used in previous studies (e.g. Enns et al. 2005; 0.1; Di Giuseppe et al. 2008: 0.08). \( \eta^{\text{yield}} \) is then added as another component in the harmonic mean expression for \( \eta^{\text{eff}} \) (eq. 1). To properly characterize the impact of varying extents of lithospheric weakening, we have also examined models with variable yield stresses for our intermediate transition strain rate model (\( \dot{\varepsilon}_{\text{p}}^{N} = 5 \times 10^{-15} \) s\(^{-1}\)). We consider plates with a range of depth-dependent yield stresses (\( \lambda = 0.1–0.2 \)), and a range of constant yield stresses (\( \tau^{\text{yield}} = 100–1000 \) MPa).

To facilitate decoupling of the SP from the OP, we insert a 15 km thick, weak (= \( \eta^{\text{mantle}} \)) crustal layer within SP (e.g. Běhouneková & Čižková 2008). The crust, defined by compositional tracers, is linearly tapered from full thickness at a depth of 200 km (and above), to zero thickness below a depth of 400 km, in order to avoid stress discontinuities. For more details about the implementation of the compositional crust, and resolution tests for similar 2-D numerical models, see Holt et al. (2015a).

### 3 RESULTS

#### 3.1 2-D Models

We now consider the effect of a power-law mantle on 2-D slabs, which are, by definition, equivalent to slabs with infinite width trenches. Numerical models which are initiated by a shallow protoslab are shown in Fig. 2. Decreasing the transition state increases the proportion of the asthenosphere deforming dominantly in the power-law viscosity regime (i.e. eq. 2). As expected, power-law deformation is focused in the portions of the asthenosphere next to the slab and surface plates (e.g. Billen & Hirth 2007; Garel et al. 2014). Here, high strain rates result from the shearing of the adjacent asthenosphere by the mobile plates/slab. Additional regions that undergo more moderate, power-law deformation are the return-flow upwellings that occur below the trailing edge of the SP, and above the slab tip. For a transition strain rate of \( 10^{-15} \) s\(^{-1}\), effective viscosities are in some places reduced by over an order of magnitude, particularly directly adjacent to the subducting slab (Fig. 2e). For a transition strain rate of \( 10^{-15} \) s\(^{-1}\), the more spatially limited power-law regions have viscosities reduced by a factor of \( 5 \), relative to the Newtonian viscosity component (Fig. 2c).

Fig. 2 shows the strong dependence of slab morphology, particularly slab dip angle, on the transition strain rate. As the transition strain rate decreases, that is, as the power-law viscosity component becomes more dominant, the slab dip angle gradually increases up until the slab bends forward over the model base for a transition strain rate of \( 10^{-15} \) s\(^{-1}\) (Fig. 2e). As discussed by Ribe (2010), whether the slab initially leans backwards or forwards at an impermeable model base is a function of the dip angle of the slab as it impinges on the boundary. For transition strain rates of \( 5 \times 10^{-15} \) s\(^{-1}\) and greater, slabs impinge on the lower boundary with a dip angle <90°, and so lean initially backwards on top of the lower boundary, while the opposite is true for a transition strain rate of \( 10^{-15} \) s\(^{-1}\).

Additionally shown in Fig. 2 are mantle flow velocities. On average, the mantle flow becomes faster as the transition strain rate is reduced. To facilitate comparison of the mantle flow regime, we therefore use a Stokes velocity scaling (\( V_{\text{Stokes}} = \Delta \rho g h^{\text{mantle}} / (\eta^{\text{mantle}}) \)) to normalize the mantle flow velocities by a reference velocity. Aside from mantle viscosity, the other \( V_{\text{Stokes}} \) components are held constant throughout this study, such that (\( \eta^{\text{mantle}} \)) values dictate our reference velocity, computed as \( V_{\text{ref}} = C / (\eta^{\text{mantle}}) \). We choose \( \eta^{\text{mantle}} \) to be the harmonic mean viscosity extracted from a sublithospheric,
vertical profile at a distance of 1000 km to the left of the trench. The reference velocity pre-factor, \( C \), is set so that \( V_{\text{ref}} \) is equivalent to the convergence rate for the appropriate (i.e. 2-D or 3-D) reference, isoviscous mantle model. Similarly, dimensional time is normalized by a reference time \( t_{\text{ref}} = 660 \text{ km} / V_{\text{ref}} \)). Aside from the model where the slab bends over forwards \( (\dot{\varepsilon}_b \approx 10^{-15} \text{ s}^{-1}) \), the large-scale mantle flow patterns are similar for the models with variable degrees of power-law deformation.

In Fig. 2(f), we plot plate velocities. To highlight variability in OP velocity which, in the forearc region, is approximately equal to the trench velocity, we normalize the plate velocities by the average SP velocity. In addition to the increasing slab dip angle, the normalized trench velocity decreases as the power-law viscosity component becomes more dominant. For a transition strain rate of \( 10^{-15} \text{ s}^{-1} \), the trench is near-stationary. The stationary trench results in only minor asthenospheric flow beneath the OP, as the subducting slab is decoupled from the mantle wedge by the relative, effective viscosity reductions associated with the power-law viscosity component.

We also examine numerical models with initial conditions that result in an equivalent backwards-bending subduction mode for all of the rheologies considered, in order to make direct comparisons (Fig. 3). In these models, slab dip also increases as the transition strain rate decreases, and the increase in slab dip is coupled with a reduction in trench retreat velocity (Fig. 3f). For equivalent models times, and \( L_{SP} = 4000 \text{ km} \), Fig. 4 shows the variation of the slab dip angle, and the log of the trench retreat velocity \( (V_T / V_p) \), as a function of the transition strain rate. Relative to the reference model with an isoviscous mantle, the dip angle increases by \( \approx 30^\circ \) and \( \log_{10}(V_T / V_p) \) decreases by \( \approx 0.33 \) in the non-Newtonian model with the lowest transition strain rate \( (10^{-15} \text{ s}^{-1}) \).

To verify the more general applicability of these results, we additionally examine suites of models with shorter initial SPs \( (L_{SP} = 2000 \text{ km}) \) and plastic yielding (eq. 3: \( \lambda = 0.15 \)). While reducing \( L_{SP} \) increases slab dip angle and reduces \( V_T / V_p \), the basic trends are unchanged (Fig. 4). Similarly, the addition of plasticity (i.e. viscosity reduction in the bending region) does not change the trend observed, but increases the relative rate of trench motion. Thus, we show that the sense of variation in slab dip and trench retreat rate, as a function of mantle rheology, is equivalent for models with both plastic yielding and short SPs. Interpretation of the models which include plastic yielding is complicated by the feedback between yielding and the degree of power-law flow in the sublithospheric mantle. As the power-law transition strain rate is reduced and the mantle moves further into the power-law regime, slab bending stresses increase which, in turn, increases the extent of the SP hinge which yields. We expect that this behaviour is responsible for the slight decrease in \( V_T / V_p \) observed at high transition strain rates (Fig. 4b). Due to this complexity, we have additionally investigated a suite of models where we hold the mantle rheology constant \( (5 \times 10^{-15} \text{ s}^{-1}) \) transition strain rate), and vary the yield stress for both Byerlee \( (\lambda = 0.1, 0.15, 0.2) \) and constant yield stress plasticity \( (\tau_{\text{yield}} = 100, 200, 1000 \text{ MPa}) \). We find that varying the yield stress yields results in changes in \( V_T / V_p \) that are of second order, relative to those induced by varying the transition strain rate. An exception occurs in the models which have yield stresses that are low enough to induce slab break-off \( (\lambda = 0.1 \text{ and } \tau_{\text{yield}} = 100 \text{ MPa}) \). Here, slab break-off exerts a strong control on plate kinematics, but this is beyond the scope of this work which is focused on mature, sustained subduction.

Figure 2. Viscosity field snapshots showing the suite of 2-D numerical models initiated with a shallow initial protoslab, at broadly equivalent time steps based on slab morphology. In addition to panels showing the models with variable transition strain rates, \( \dot{\varepsilon}_b \) (c–e), the equivalent isoviscous mantle model (b), and initial conditions (a), are shown. Overlain are mantle velocity vectors normalized by a reference velocity, computed as described in the main text. The normalized model times (bottom right corner) are computed by dividing the dimensional times by a reference time \( (t' = t / t_{\text{ref}} = V_{\text{ref}} / h) \). Inverted triangles indicate the initial trench location. The bottom panel, (f), compares the plate velocities, normalized by the average SP velocity.
Figure 3. As in Fig. 2, but for the 2-D models with a deeper initial slab geometry.

**Figure 4.** Plate kinematics as a function of transition strain rate for the 2-D models with deep initial slab geometry, extracted at time steps equivalent to those shown in Fig. 3. In addition to the long SP models shown in Fig. 3 (initial $L_{SP} = 4000$ km: black, solid lines), kinematics from short SP models ($L_{SP} = 2000$ km: black, dashed lines), and long SP models with plasticity ($L_{SP} = 4000$ km and $\lambda = 0.15$: red lines) are plotted. The plasticity suite of models are plotted at an earlier stage of subduction, but still after flattening on the model base, due to later-stage instability (i.e. OP subducts) for the lowest transition strain rate model. Shown is, (a), the average slab dip angle computed between depths of 200 and 460 km and, (b), the log of the ratio of trench retreat velocity and SP velocity. The kinematics extracted from the equivalent Newtonian models are placed at an arbitrary, ‘infinite’ transition strain rate x-location, for illustrative purposes.

3.2 3-D Models

We now present 3-D numerical models to explore whether the 2-D results are robust in 3-D. Additionally, we probe the effect that such a non-linear rheology has on 3-D mantle flow.

3.2.1 Model evolution

We analyse 3-D models with equivalent properties, and initial conditions, to the suite of 2-D models shown in Fig. 3. Fig. 5 shows the viscosity fields for models both with an isoviscous mantle, and with a composite power-law rheology ($\dot{\varepsilon}_T^0 = 5 \times 10^{-15}$ s$^{-1}$). As in the 2-D models, the dip angle is elevated in the power-law models, and the power-law regime is active primarily in the portion of the asthenosphere being directly sheared by adjacent plates/slab. However, in the 3-D models, a significant portion of the subduction-induced mantle return flow occurs in the form of toroidal flow around the slab edge, which induces strain rates large enough (i.e. $>5 \times 10^{-15}$ s$^{-1}$) for deformation to occur within the power-law regime. We have additionally examined equivalent models where the OP is absent (Fig. 6), and where the base of the model has a no slip boundary condition (not shown). In the single slab experiments, the lack of an
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Figure 5. Comparison of viscosity and mantle flow fields for 3-D models ($L_{SP} = 4000$ km) with, (a), an isoviscous mantle and, (b), a composite power-law/Newtonian mantle rheology with a transition strain rate of $5 \times 10^{-15} \text{s}^{-1}$. Both models are plotted at broadly equivalent time steps based on slab morphology. For each model the upper panel shows a horizontal, sublithospheric slice ($z = 200$ km), and the lower panel shows a vertical slice through the model/plate mid-plane ($y = 0$). In the horizontal slices, the surface plate geometries are overlain in black.

OP promotes both the development of slab curvature and slab/trench rollback (e.g. Stegman et al. 2006; Schellart et al. 2007), relative to the models which include an OP. Therefore, power-law deformation associated with rollback-driven toroidal flow is greater in the single slab model (cf. Figs 5b and 6b). In the model with a no slip base, the subducting slab morphology is similar to that observed in the free slip models, but subduction rates are reduced by 30–40 per cent, and the size of regions dominated by power-law deformation is also reduced.

As for the 2-D cases, we consider models with variable initial SP lengths. While the power-law deformation is generally located preferentially below the SP, a shift towards more wide-spread power-law deformation below the OP occurs for greater $L_{SP}$. This occurs because as $L_{SP}$ increases, the OP moves more rapidly due to trench retreat, which results in greater shear strain rates below the OP. Where power-law deformation occurs within the sublithospheric mantle dictates the spatial distribution of basal traction on the lithospheric plates which, in turn, dictates the overall effect of a power-law mantle on slab kinematics. This is discussed in detail in section 4.3.

3.2.2 3-D mantle flow

We now compare the 3-D mantle flow field for the long SP models with, and without, a power-law viscosity component ($\dot{\epsilon}_{II} = 5 \times 10^{-15} \text{s}^{-1}$). The vertical and horizontal mantle flow field, normalized as described in Section 3.1, is shown in Fig. 7. Prior to normalization, the average mantle flow velocity is $\approx 50$ per cent greater in the power-law models, due to reduced average mantle viscosity. However, after normalizing, the flow patterns are again very similar for the two models. Similarly, for models both without an OP, and with no slip basal boundary conditions, we find that a
power-law mantle rheology also exerts only a minor influence on the broad scale mantle flow pattern. As subduction is partitioned between trench retreat and SP advance in these models, return flow is partitioned between, respectively, toroidal flow around the slab edges and upwelling in the vertical plane. In all of the models explored, there are multiple regions with significant upwards-directed vertical mantle flow, the locations of which may have implications for the location of decompression melting-induced volcanism (e.g. Faccenna et al. 2010). In addition to upwelling flow at the trailing edges of the two plates, a vigorous mantle upwelling occurs at the lateral edge of the slab-tip, and more moderate vertical flow is associated with flow around the slab edge (i.e. subhorizontal toroidal flow cell).

In order to quantify the mantle flow characteristics, we examine the mantle flow partitioning between toroidal and poloidal components (e.g., Bercovici 1993). Following, for example, Tackley (2000), the 2-D velocities on each horizontal layer are decomposed into irrotational (i.e. poloidal) and divergence-free (i.e. toroidal) vector fields. The vertical velocity field is added to the poloidal field, and toroidal/poloidal ratio (TPR) is taken as the toroidal RMS velocity divided by the poloidal RMS velocity. We compute the RMS of the poloidal and toroidal velocity fields for 10 horizontal layers ($\Delta z = 66$ km), and plot vertical profiles of the horizontal RMS velocities in order to characterize both the role of an OP (Fig. 8), and a power-law mantle rheology (Fig. 9), on mantle flow partitioning.

We examine the flow field for our two plate length end-members, $L_{SP} = 2000$ and 4000 km (Fig. 8). As discussed, trench retreat is dominant in the $L_{SP} = 4000$ km model, for slabs both with and without OPs. This results in the dominance of trench retreat-induced toroidal flow (i.e. $\log_{10}(TPR) > 0$) over a substantial depth range ($200 \text{ km} < z < 460 \text{ km}$) (Fig. 8c). Toroidal flow around the slab edge is focused in the mid-upper mantle, while poloidal mantle flow is dominant only near both the surface and the model base (cf. Becker & Faccenna 2009; Fig. 10). In contrast, the shorter SP model exhibits dominantly poloidal flow at all depths (i.e. $\log_{10}(TPR) < 0$). The absence of an OP enhances the relative importance of trench retreat, thereby increasing TPR at mid-mantle depths. This effect is

Figure 6. As in Fig. 5, but for the models without an OP.
particular prominent for the short SP case, in which the absence of a large OP exerts a stronger effect on slab dynamics than in the long SP models. This can be seen in the lower panels of Fig. 8, where much stronger variability in slab rollback is evident for the short SP models, relative to the long SP models (cf. Figs 8d and e).

In line with the qualitative mantle flow similarity observed in Fig. 7, the inclusion of a power-law mantle has only a moderate effect on the TPR (Fig. 9). For the short SP model, a power-law mantle rheology reduces $\log_{10}(\text{TPR})$ by 0.05–0.1, and for the long SP model, the TPR is approximately equivalent (slightly elevated). As observed in the 2-D models, a power-law mantle rheology decreases trench retreat rate, and so we suggest that the reduction in TPR, for the short SP model, stems from the reduction in relative retreat rate. However, for the long SP model, the relative reduction in trench retreat velocity does not have such an expected effect. We propose that, in addition to the effect that reducing trench retreat has by a power-law viscosity promote toroidal flow. As discussed in the next section, a power-law viscosity component has a reduced effect on trench motions for the long SP models (Fig. 10c), and so one may expect the lateral viscosity variation effect to dominate here.

3.2.3 Slab kinematics
As for the 2-D models presented in section 3.1, we find that a power-law mantle rheology reduces the magnitude of trench retreat, $V_T$, relative to SP velocity, $V_P$. This is true for each of the SP lengths examined, as shown in Fig. 10 where we plot the temporal evolution of the normalized plate velocities, in the side plate reference frame. Generally, $V_P$ is greater than $V_T$ (i.e. negative log ($V_T / V_P$): Fig. 10c), with the only exception being the isoviscous mantle, long SP model. After the initial, transient bending phase, both plate velocities generally attain a near-constant,
steady-state value. As discussed previously, as \( L_{SP} \) increases, plate convergence is increasingly accommodated by trench retreat (i.e. \( V_T/V_P \) increases). For all plate lengths, the normalized \( V_T \) of the power-law rheology models (dashed lines) is generally comparable to the isoviscous mantle models (solid lines). This similarity occurs because the average viscosity used to compute \( V_{ref} \) is extracted from SP side of the slab (i.e. the portion of the mantle which dictates \( V_T \)). In contrast, the power-law mantle viscosity models exhibit a strong reduction in \( V_T \) (Fig. 10b), for all plate geometries. This can be seen in Fig. 10(c), where the temporal evolution of the log of the ratio of \( V_T \) and \( V_P \) is plotted. For every plate length, \( \log \frac{V_T}{V_P} \) is consistently reduced in the power-law mantle models. The relative reduction in \( V_T \) reduces as the SP length increases (for \( L_{SP} = 2000 \) km, \( \Delta \log \frac{V_T}{V_P} \approx 0.15 \); for \( L_{SP} = 4000 \) km, \( \Delta \log \frac{V_T}{V_P} \approx 0.07 \)).

We additionally examine isoviscous mantle models with average viscosities equivalent to those observed in the power-law models (as extracted from viscosity profiles taken below the SP). This is done to examine whether the localized viscosity reductions, or the bulk mantle viscosity reduction (which reduces the plate-mantle viscosity contrast), are responsible for the relative \( V_T \) reduction. Reducing the mantle viscosity in the Newtonian models does not produce the reduced \( V_T/V_P \) observed in the power-law mantle models (Fig. 10c, lighter coloured solid lines). We therefore attribute the reduction in relative trench motions to the localized viscosity reductions that occur directly adjacent to the lithospheric plates in the power-law models.

As in the 2-D models (Fig. 4a), the dip angle increases as power-law deformation becomes more dominant (i.e. as transition strain rate decreases). For the long SP models, Fig. 11 illustrates the dependence of \( V_T/V_P \) and the, inversely correlated, slab dip on the transition strain rate. For transition strain rates of \( 5 \times 10^{-15} \) s\(^{-1} \), and above, the mantle is almost entirely in the Newtonian regime, and so the slab kinematics are nearly equivalent to those observed in the isoviscous mantle model (Fig. 11). While the sense of kinematic variability is equivalent for the 2-D and 3-D models, the magnitude of variation is not. In 3-D, \( \log_{10}(V_T/V_P) \) is reduced by 0.09 for a model with a power-law transition strain rate of \( 5 \times 10^{-15} \) s\(^{-1} \) (relative to the isoviscous mantle model). The equivalent 2-D models produce a significantly larger reduction of 0.19 (Fig. 4b).
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Figure 9. Partitioning of mantle flow between poloidal and toroidal components, for models with an isoviscous mantle (solid lines) and with a power-law rheology with a transition strain rate of $5 \times 10^{-15}$ s$^{-1}$ (dashed lines). Two initial SP length end-members are plotted (black: $L_{SP} = 2000$ km, red: 4000 km). Shown are vertical profiles, which are composed of velocities averaged over 10 horizontal slices ($\Delta z = 66$ km). Plotted is, (a), RMS of poloidal velocity, (b), RMS of toroidal velocity and, (c), the log ratio of the toroidal to total poloidal RMS velocities. Velocities are extracted at the time steps plotted in Figs 5 and 7, and normalized by a reference velocity as described in the main text.

Figure 10. Temporal evolution of plate kinematics for isoviscous mantle (solid lines), and power-law mantle (dashed lines: $\dot{\varepsilon}_{II} = 5 \times 10^{-15}$ s$^{-1}$) models, with variable initial SP lengths. Panels show, (a), the SP velocity (side plate / box reference frame), (b), the trench retreat velocity and, (c), the log of the trench retreat velocity divided by the SP velocity. The lighter, solid lines, on panel (c), display the log ratio for additional Newtonian models which have an average plate–mantle viscosity contrast that is set to be equivalent to that of the power-law models with the same $L_{SP}$. Plate velocities are normalized by reference velocities computed using sublithospheric viscosities extracted at the time-steps indicated by points. The reference velocity pre-factor is set to give $V_{ref} = V_C$ for the medium plate length (2000 km), isoviscous mantle model. Grey shading indicates the transient phase, where the slab is bending on top of the lower boundary.

Laboratory-derived olivine flow laws tend to observe dislocation creep activation in approximately the same regions as in our models, particularly directly below the lithospheric plates and adjacent to the sinking slab (e.g. Billen & Hirth 2005; Čížková & Bina 2013; Billen & Hirth 2007; Androvičová et al. 2013; Garel et al. 2014). In addition to broadly matching the depth extent of dislocation creep inferred from seismic anisotropy (see Section 2.1.1), this similarity gives us confidence that our models capture the first order dynamic effects associated with dislocation creep in the upper mantle, provided these models use the ‘correct’ grain size. As explored in
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previous work, primary effects of viscosity reductions adjacent to the subducting slab, and in the mantle wedge, include elevated slab sinking/convergence rates, and greater slab dip angles (e.g. Christensen et al. 2006; Holt et al. 2015a).

As discussed by Billen & Hirth (2005, 2007), this slab dip increase occurs in part due to relative viscosity reductions in the mantle wedge which reduce the magnitude of negative pressure, and so the associated slab-lifting torque (Stevenson & Turner 1977; Tovish et al. 1978). This reduction in across-slab dynamic pressure gradient (i.e. $\Delta P_{\text{linear}} - \Delta P_{\text{power-law}}$) is, averaged over the various plate lengths, $\sim$5 MPa. For an average dip angle of $\sim$70°, such a reduction in $\Delta P$ produces a reduction in the lifting force, $F_{\Delta P}$, of $\approx$3.5 TN m$^{-1}$ (for our models, $F_{\text{slab pull}} \approx$ 44 TN m$^{-1}$). Billen & Hirth (2005, 2007) hold the trench position fixed and so only consider the role of a reduced $\Delta P$ on slab dip angle. However, dip angle is also strongly affected by trench motion, $V_T$ (e.g. Christensen 1996; Ribe 2010). In our numerical models, we find that a power-law mantle viscosity reduces the relative magnitude of trench retreat, which consequently also promotes greater slab dip angles. We, therefore, suggest that allowing the trench to migrate freely will enhance the slab dip increasing effect of dislocation creep. However, a reduction in $F_{\Delta P}$, which has a horizontal component directed towards the mantle wedge, constitutes a reduction in one of the major forces resisting trench retreat (e.g. Royden & Husson 2006; Stegman et al. 2006), and so understanding why slabs subducting into a power-law mantle exhibit reduced $V_T / V_P$ requires an analysis of the additional forces associated with mantle flow (Section 4.3).

4.2 Mantle flow and 3-D effects

We have additionally examined the impact of a power-law mantle viscosity on 3-D mantle flow. In our numerical models, we find that a power-law rheology does not significantly impact the mantle flow patterns (cf. Christensen 1984), nor the partitioning of flow between poloidal and toroidal components (cf. Čádek et al. 1993; Becker 2006). When assessing the magnitude of the subduction-resisting forces associated with mantle flow (Section 4.3, Fig. 13), such mantle flow similarity is convenient in that it allows similar expressions to be used to compute mantle shear stresses for both rheologies.

We suggest that the lack of variation in mantle flow partitioning is due to competing power-law effects, namely reduced rollback rates and increased lateral viscosity variations. However, it could also be the case that the transition strain rate imposed in our models, which controls the degree of power-law deformation, is still too high to generate the full extent of dislocation creep active on Earth. Previous numerical modelling studies, which utilize laboratory derived rheologies, report mantle flow velocities that are up to a factor of 8 greater than plate velocities, particularly within the mantle wedge (Billen & Jadamec 2012; Jadamec & Billen 2012). In our study, the relative sublithospheric mantle flow velocities (i.e. normalized by the plate velocity) are typically only 5–10 per cent greater for the reference power-law, composite rheology model (Figs 12a and c). However, a direct comparison is complicated by the complex subduction geometry, and strong lithospheric plates (on average, four orders of magnitude stronger than the asthenosphere) considered in the studies mentioned above. Also, these studies compute instantaneous flow and so it is not obvious that stable subduction would occur for time-dependent versions of these setups. A combination of these two modelling strategies, that is, dynamically evolving 3-D subduction and a laboratory-derived rheological parameterization, is therefore required to further assess the impact of dislocation creep on subduction-induced mantle flow.

In all of the models explored, there are multiple regions with significant upwards-directed, vertical mantle flow. In addition to upwelling flow at the trailing edges of the two plates, a vigorous mantle upwelling occurs at the lateral edge of the slab-tip, and more moderate vertical flow is associated with flow around the slab edge (i.e. subhorizontal toroidal flow cell). These flow features are consistent with the study of Facenna et al. (2010), where these types of upwellings are observed in single slab numerical models, and invoked to explain the anomalous off-arc volcanism observed in locations such as the western US and the Mediterranean, and the analogue modelling studies of Schellart (2010) and Strak & Schellart (2014). Our models verify the presence of such upwellings in dynamic subduction models that incorporate both an OP, and a stress-dependent mantle rheology. Additionally, beneath the OP and $\sim$500 km away from the trench, we observe moderate power-law deformation which could have potential implications for melt/volatile transportation in the mantle wedge/forearc region.

Relative to the 2-D models, we find that the magnitude of variation of both slab dip and trench retreat rates induced by variable mantle rheology is significantly reduced in 3-D. In 3-D, the presence of a finite width trench enables toroidal mantle flow around the slab edge, which can act to relieve subslab pressures, thereby enabling rapid trench retreat (e.g. Fucinelli et al. 2003; Schellart 2004; Royden & Husson 2006; Stegman et al. 2006, 2010). However, in 2-D, such a mechanism for relieving elevated subduction pressures is absent. Therefore, the variation in across-slab pressure gradient induced by mantle rheology is likely to be greater in 2-D, and thus require greater variation in the slab kinematics to accommodate this (i.e. satisfy the appropriate force balances). The presence of a viscous lower mantle, neglected here for simplicity, may reduce the magnitude of this 2-D effect by enabling subslab pressure build-up to be relieved by deep, poloidal flow (e.g. Husson et al. 2012).

4.3 A mechanism for reduced trench retreat

A simplified analysis of the magnitudes of the forces resisting subduction may provide further insights into why a power-law asthenospheric viscosity component promotes subduction to occur.
preferentially via SP motion, $V_p$, as opposed to trench retreat, $V_T$. Previous studies have utilized adaptations of the expression derived by Conrad & Hager (1999), which consists of a balance between slab pull and the subduction-resisting forces (i.e., slab bending, mantle drag, and interfacial coupling), to predict convergence rates ($V_C$) for a range of lithospheric properties (e.g., Funiciello et al. 2003; Bellahsen et al. 2005). Here, we instead compute approximations of the forces resisting both trench retreat and SP motion, for our observed $V_C$ and across-slab pressure gradients ($\Delta P$). We focus on the forces associated with mantle flow, as these are the force components directly modified by the presence of a power-law rheology. Therefore, we do not explicitly consider slab pull and slab bending which, respectively, drive and resist subduction, in an approach similar to that of Schmeling et al. (2005).

While the inclusion of a power-law viscosity component significantly modifies the mantle viscosity (e.g., Fig. 5), the style of subduction-induced mantle return flow is comparable for models with and without such a mantle rheology (Fig. 7). We therefore assume that the ratios between the plate/slab velocities and the adjacent mantle velocities are equivalent for both mantle rheologies. We compute the plate/mantle velocity ratios from the intermediate SP length model ($L_{SP} = 3000$ km), and then simply use the value averaged over both rheologies for the subsequent shear force calculations. Fig. 12 shows the mantle velocity profiles used for this calculation and verifies the similarity between the flow profiles for the two models. Beneath the SP, the asthenosphere is flowing slower than the plate, and so exerts basal tractions which resist SP motion, $V_p$ (Fig. 12a). Conversely, the subduction-induced flow beneath the OP is faster than the plate motion which results in basal tractions that are driving trench-wards OP motion (Fig. 12c). The subduction-resisting shear force per unit length on the SP is given by the product of an average sublithospheric shear stress and the SP length,

$$F_{SP} = (2\eta_{m,SP} \dot{\epsilon}_{\text{shear}})X_{SP} \approx \eta_{m,SP} \left( \frac{V_p - V_{m,p}}{\Delta z} \right) X_{SP},$$

where $V_p$ and $V_{m,p}$ are, respectively, the SP velocity and the horizontal velocity of sublithospheric mantle, and $\Delta z$ is the depth that $V_{m,p}$ is extracted at, relative to the base of the lithospheric plate (e.g., Fig. 12a). This assumes that the horizontal gradient of vertical velocity is negligible, as can be seen from the mantle flow field (e.g., Fig. 5). $\eta_{m,SP}$ is the average sublithospheric viscosity beneath the SP, and $X_{SP}$ is the SP length over which this average shear stress acts ($\approx L_{SP} - 1000$ km). Similarly, the OP motion-driving force is given by,

$$F_{OP} \approx \eta_{m,OP} \left( \frac{V_C - V_p - V_{m,OP}}{\Delta z} \right) X_{OP},$$

where $V_C$ is the convergence rate ($= V_T + V_p$), and $X_{OP}$ is the approximate OP length over which this average shear stress acts. $X_{OP}$ does not appear to have a dependence on plate length and so, based on inspection of Fig. 12 and similar figures with flow profiles plotted at different distances from the slab, we use an average $X_{OP}$ of 1200 km. In addition we compute the approximate, up-dip directed shear force, $F_{slab}$, acting on the slab,

$$F_{slab} \approx 2 \eta_{m,slab} \left( \frac{V_C - V_{m,slab}}{\Delta n} \right) \frac{h}{\sin(\theta)},$$

where $V_{m,slab}$ is the slab-parallel mantle flow velocity at a slab-normal distance of $\Delta n$ away from the slab edge, $h$ is the depth-extent of the slab ($= 660$ km $- h_1 = 580$ km), and $\theta$ is the dip angle. The additional factor 2 occurs due to mantle shear occurring on both sides of the slab. In the power-law numerical models, mantle viscosity is a function of the deviatoric strain rate, $\dot{\varepsilon}_H$ (i.e., eq. 2). Under the assumption that the only non-zero strain rate component is the shear strain rate (in the vertical plane), $\dot{\varepsilon}_H = \dot{\varepsilon}_{\text{shear}}$; and so, for example, the power-law component of the sublithospheric viscosity below the SP can be written as

$$\eta_{m,SP}^\prime = \eta_{m,SP} \left( \frac{V_p - V_{m,p}}{2 \Delta z \dot{\varepsilon}_H} \right)^{1/n},$$

which is then combined with $\eta_{m,slab}$ to get the effective, sublithospheric viscosity (eq. 1), and inserted into the expression for $F_{SP}$ (eq. 4). An approximation for the total, mantle force resisting SP motion, $F_p$, is taken to be the sum of the subduction-resisting shear forces associated with the flat-lying portion of the SP and the slab, $F_p = F_{SP} + F_{slab}$. Similarly, the approximate, mantle-derived, force resisting trench retreat is composed of a component associated with an along-slab mantle pressure gradient, $\Delta P$, and the force driving OP motion, i.e., $F_T = F_{OP} - F_{OP}$. The directions of each of these forces considered are shown on Fig. 12(a). Within this $F_T$ is the assumption that $F_{OP}$—the horizontal mantle shear force on the OP—is transmitted to the SP in the form of a horizontal force promoting rollback (i.e., by ‘pushing’ the slab hinge, e.g., van
Dinther et al. (2010). While a proportion of $F_{SR}$ is likely dissipated in the weak plate boundary, we neglect this in order to simplify our calculations. The other component of the force resisting trench retreat, the horizontal component of $F_{SR}$, is computed using the average pressure gradient extracted from our numerical models ($F_{SR} = \Delta P(h/\sin(\theta))$). $\Delta P$ is computed using two points 330 km either side of the slab, at mid-mantle depths ($z = 330$ km).

We consider a power-law rheology equivalent to that implemented in the numerical models, with a transition strain rate of $5 \times 10^{-15}$ s$^{-1}$. As input into our simplified model we use the average $\theta$, $V_C$, and $\Delta P$ observed in the isoviscous ($65^\circ$, 6.9 cm yr$^{-1}$, 18 MPa) and power-law mantle viscosity models ($75^\circ$, 10.3 cm yr$^{-1}$, 13 MPa), but ignore variation associated with the variable plate lengths in order to keep the calculations general. We do however include the average variability because the power-law reduction in $\Delta P$ and increase in $V_C$ both promote trench retreat (in our calculation), and so we wanted to determine whether, despite this, the relative force resisting trench motion would increase.

Fig. 13 shows the magnitude of the calculated trench resisting ($F_{T}$) and plate motion resisting forces ($F_{P}$), normalized by the slab pull force ($= 44$ TN m$^{-1}$), as a function of relative trench motion, $V_T / V_C$ (i.e. $= 0$ = pure plate motion, $1$ = pure trench motion). For a fixed trench, $F_{T}$ is composed solely of the resisting force associated with mantle pressure (Fig. 13(a)). With increasing trench motion, the trench retreat resisting force reduces, as a result of an increasing $F_{SR}$ which acts to promote retreat (eq. 5). The magnitude of this reduction in $F_{T}$ is lower when a power-law rheology is considered, as $F_{SR}$ reduces due to the relative reduction in asthenospheric viscosity. The force resisting plate motion, $F_{P}$, is shown in Fig. 13(b).

Generally, $F_p$ is reduced for the power-law models. This occurs because the magnitude of the subduction-resisting shear tractions associated with both the flat-lying portion of the SP ($F_{SR}$), and the slab ($F_{slab}$), are reduced when a power-law viscosity is considered. Additionally, models with longer SPs have elevated $F_{P}$ due to the effect of increasing $X_{SR}$ on $F_{SR}$, and the curves have negative trends due to the dependence of $F_{SR}$ on $V_{SP}$ (eq. 4).

We plot the ratio of the trench retreat resisting force and the plate motion resisting force, $F_T / F_P$ (Fig. 13c). Also included on Fig. 13(c) are points indicating the $V_T / V_C$ values observed in our numerical models, and the present day, median $V_T / V_C$ values observed on Earth (Heuret & Lallemand 2005), for a range of APF reference frames. As the trenches in our numerical models are, by design, retreating, we consider only retreating trenches for the plotted Earth $V_T / V_C$ averages. However, we note that the $V_T / V_C$ range is comparable when advancing trenches are included (i.e. with advancing trenches: $V_T / V_C = 0.32-0.55$; without advancing trenches: 0.33–0.58). Our models generally lie within these kinematic ranges, with the exception of the short SP models ($L_{SP} = 2000$ km), in which $V_T$ is overly dominant (Fig. 13c).

We consider $F_T / F_P$ to be an approximate measure of the degree to which the mantle forces favour trench retreat (i.e. low values) or plate motion. This measure does not consider other important forces, both driving (i.e. slab pull) and resisting (e.g. slab bending), in the system, and so only constitutes an approximate metric for plate versus trench motion. However, it does mirror the first-order features observed in the slab kinematics (Fig. 10c), suggesting that mantle-related forces exert an important control on slab kinematics (cf. Schmeling et al. 2005). Firstly, plate motion is increasingly favoured as $L_{SP}$ decreases, as observed in the numerical models. This occurs primarily because the plate motion resisting force, $F_{SP}$, reduces as $L_{SP}$ decreases. More importantly, $F_T / F_P$ is elevated for the power-law models for all plate lengths. This consistent increase in the relative $F_T$ is a robust feature of the analysis, regardless of whether we include individual model values for $\Delta P$ and $\theta$ (i.e. no averaging), or use the same values for all models (i.e. increased averaging). This occurs because the localized viscosity reductions induced by the power-law rheology reduce the mantle shear resistance to both horizontal plate sliding ($F_{SP}$) and slab sinking ($F_{slab}$), to a greater degree than a reduction in mantle wedge pressure ($F_{SR}$) reduces the mantle resistance to trench retreat. We therefore suggest that it is these effects which preferentially promote SP motion, over trench retreat, for the models with a power-law mantle viscosity component.

Our numerical models with reduced, bulk mantle viscosities do not produce such a reduction in relative trench retreat rate (Fig. 10c), and so we emphasize that this is an effect of relative viscosity
reductions adjacent to the lithospheric plates. This constitutes an example of reduced sublithospheric shear stresses shifting the partitioning of plate convergence between plate and trench motion. While we focus here on power-law deformation, such localized viscosity reductions could potentially also occur due to volatile enrichment and/or mantle melting.

4.4 Model and analysis limitations

While this force analysis has enabled us to probe the mantle forces that control the partitioning of subduction between rollback and plate advance for our observed kinematics, it does not allow us to directly predict $V_T / V_P$. Full force balances, which include the additional important forces (i.e. slab pull, slab bending, plate interface coupling) (e.g. Royden & Husson 2006; Billen 2008), are required to dynamically compute $V_C$ and $\Delta P$, and so in turn assess the magnitude of this effect for various subduction setups (e.g., lithospheric and mantle strengths). However, simply extracting these components from the numerical models is sufficient to illustrate our proposed mechanism for trench retreat reduction.

Our force assessment focuses on the variation in mantle forcing as a function of mantle rheology, thereby neglecting forces directly associated with the lithospheric plates. This is reasonable as slab density, viscosity, and thickness are fixed throughout this study. An exception, and potential source of lithospheric-induced force variation, is the variation of slab radius of curvature, $R$, which exerts a strong control on the slab bending force for a viscous slab $F_B \propto \eta \gamma v_c (h_t / R)^3$ (Conrad & Hager 1999), as a function of both mantle rheology and time (e.g. Becker et al. 1999; Buffett 2006; Stegman et al. 2006; Capitano et al. 2009). Further work is therefore required to assess potential feedbacks between slab curvature and a stress dependent mantle rheology, in order to untangle this component of the slab force balance and determine its expected kinematic expression.

While a simplified setup has allowed us to extract the first order effects of a power-law mantle rheology on slab kinematics, we suggest that both the lithospheric rheology and the properties of the lower mantle may play important roles. The rate of trench retreat and the degree of subducing slab weakening, in the form of plastic yielding or a maximum viscosity cut-off, have been demonstrated to exert first order controls on the propensity of a slab to penetrate into a strong lower mantle (e.g. Zhong & Gurnis 1995; Christensen 1996; Čižkova et al. 2002; Enns et al. 2005; Billen 2008). Future work focused on understanding how slab impingement upon a strong lower mantle influences the rollback systematics observed here is required to refine our understanding of trench retreat rate reduction due to power-law mantle deformation.

5 CONCLUSIONS

We used 3-D, dynamic models of subduction to address the role of a power-law mantle viscosity on rates of trench retreat. For a suite of models that exhibit a broad range of slab kinematics, we find that a power-law rheology results in reduced rates of trench retreat, $V_T$, and elevated slab dip angles, relative to the equivalent isoviscous mantle models. We analyse the style of mantle flow, which exhibits only limited variability as a function of mantle rheology, and mantle pressure, in order to compute estimates of the forces resisting both $V_T$, and SP advance, $V_P$. We find that the inclusion of a power-law rheology reduces the mantle shear force, which resists $V_T$, to a greater extent than it reduces the dynamic pressure gradient across the slab, which resists $V_P$. Thus, the inclusion of a power-law mantle rheology favours a shift towards a subduction mode with a reduced trench retreat component. In addition to the inclusion of an OP, we suggest that this mechanism may be of key importance in reducing the high relative $V_T$ rates often observed in single slab models to levels more in line with the average subduction partitioning observed on Earth (i.e. $V_T \leq V_P$). More sophisticated numerical models, and analytical force balances, are required to more rigorously map out the ‘Earth-like’ parameter space where this mechanism is viable.

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